

The generalized second law of gravitational thermodynamics on the apparent horizon in $f(R)$ -gravity

K. Karami^{1,2*}, M.S. Khaledian^{1†}, N. Abdollahi¹

¹Department of Physics, University of Kurdistan, Pasdaran St., Sanandaj, Iran

²Research Institute for Astronomy & Astrophysics of Maragha (RIAAM), Maragha, Iran

April 17, 2012

Abstract

We investigate the generalized second law (GSL) of thermodynamics in the framework of $f(R)$ -gravity. We consider a FRW universe filled only with ordinary matter enclosed by the dynamical apparent horizon with the Hawking temperature. For a viable modified gravity model as $f(R) = R - \alpha/R + \beta R^2$, we examine the validity of the GSL during the early inflation and late acceleration eras. Our results show that for the selected $f(R)$ -gravity model minimally coupled with matter, the GSL in the early inflation epoch is satisfied only for the special range of the equation of state parameter of the matter. But in the late acceleration regime, the GSL is always respected.

PACS numbers: 04.50.Kd

Keywords: Modified theories of gravity

*KKarami@uok.ac.ir

†MS.Khaledian@uok.ac.ir

1 Introduction

In the last decade, the cosmological observations have confirmed the existence of the early inflationary epoch and the accelerated expansion of the present universe [1]. The proposals that have been put forth to explain these interesting discoveries can basically be classified into two categories. One is to assume the existence of an exotic energy with negative pressure, named dark energy (DE). However, the problem of DE is one of the hardest and unresolved problems in modern theoretical physics (see [2, 3] and references therein).

Another alternative is to modify Einstein's general relativity (GR) theory, named modified gravity. One such modification is the $f(R)$ theory, where the Ricci scalar R in the Einstein-Hilbert action is generalized to an arbitrary function f of R (for a good review see [4] and references therein). There are also some other classes of modified gravities containing $f(\mathcal{G})$, $f(R, \mathcal{G})$ and $f(T)$ which are considered as gravitational alternatives for DE [5, 6, 7, 8, 9, 10]. Here, $\mathcal{G} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$ is the Gauss-Bonnet invariant term. Also $R_{\mu\nu\rho\sigma}$ and $R_{\mu\nu}$ are the Riemann and Ricci tensors, respectively, and T is the torsion scalar. The modified gravity can explain naturally the unification of earlier and later cosmological epochs without resorting to the DE [4]. Moreover, modified gravity may serve as dark matter (DM) [11].

Thermodynamical interpretation of gravity is one of the other interesting issues in modern cosmology. It was shown that the differential form of the Friedmann equation in the Einstein gravity can be written in the form of the first law of thermodynamics (Clausius relation) $-dE = T_A dS_A$ on the apparent horizon \tilde{r}_A . Here, $T_A = 1/(2\pi\tilde{r}_A)$ and $S_A = \frac{A}{4G}$ are the Hawking temperature and entropy on the apparent horizon, respectively, and A is the area of the horizon [12]. Also dE is the amount of the energy flow through the fixed apparent horizon. Investigation on the deep connection between gravity and thermodynamics has been also extended to some modified gravity theories like the $f(\mathcal{G})$ theory [12], scalar-tensor gravity and $f(R)$ -gravity [13], Lovelock theory [14] and braneworld scenarios (such as DGP, RSI and RSII) [15].

Note that in thermodynamics of apparent horizon in the Einstein gravity, the geometric entropy is assumed to be proportional to its horizon area, $S_A = \frac{A}{4G}$ [12]. However, this definition for the other modified gravity theories is changed. For instance, the geometric entropy in $f(R)$ -gravity is given by $S_A = \frac{Af_R}{4G}$ [16], where the subscript R denotes a derivative with respect to the curvature scalar R . In $f(T)$ -gravity, it was shown that when f_{TT} is small, the first law of black hole thermodynamics is satisfied approximatively and the entropy of horizon is $S_A = \frac{Af_T}{4G}$ [17], where subscript T denotes a derivative with respect to the torsion scalar T .

Besides the first law, the GSL of gravitational thermodynamics in the accelerating universe driven by the DE or due to the modified gravity has been studied extensively in [18]-[26]. The GSL of thermodynamics like the first law is a universal principle governing the universe.

Here, our aim is to investigate the GSL of thermodynamics in the framework of $f(R)$ -gravity for a Friedmann-Robertson-Walker (FRW) universe filled with the ordinary matter. To do this, in section 2, we briefly review the $f(R)$ -gravity. In section 3, we investigate the GSL of thermodynamics on the dynamical apparent horizon with the Hawking temperature. In section 4, we examine the validity of the GSL for a viable $f(R)$ model. Section 5 is devoted to conclusions.

2 $f(R)$ -gravity

In $f(R)$ -gravity, the modified Einstein-Hilbert action in the Jordan frame is given by [4]

$$S_J = \frac{1}{2k^2} \int d^4x \sqrt{-g} f(R) + S_m, \quad (1)$$

where $k^2 = 8\pi G$. Also g and S_m are the determinant of metric $g_{\mu\nu}$ and the matter action, respectively.

Taking variation of the action (1) with respect to $g_{\mu\nu}$ leads to the corresponding field equations in $f(R)$ -gravity

$$R_{\mu\nu}f_R - \frac{1}{2}g_{\mu\nu}f - \nabla_\mu\nabla_\nu f_R + g_{\mu\nu}\nabla^2 f_R = k^2 T_{\mu\nu}^{(m)}, \quad (2)$$

where $T_\nu^\mu = \text{diag}(-\rho_m, p_m, p_m, p_m)$ is the matter energy-momentum tensor in the perfect fluid form.

Now we consider a spatially non-flat universe described by the FRW metric

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - Kr^2} + r^2 d\Omega^2 \right), \quad (3)$$

where $K = 0, 1, -1$ represent a flat, closed and open universe, respectively. Substituting the FRW metric (3) in the field equations (2) yields the Friedmann equations in $f(R)$ -gravity as

$$k^2 \rho_m = \frac{f}{2} - 3 \left(\dot{H} + H^2 - H \frac{d}{dt} \right) f_R, \quad (4)$$

$$k^2 p_m = -\frac{f}{2} + \left(\dot{H} + 3H^2 + \frac{2K}{a^2} - 2H \frac{d}{dt} - \frac{d^2}{dt^2} \right) f_R, \quad (5)$$

where

$$R = 6 \left(\dot{H} + 2H^2 + \frac{K}{a^2} \right), \quad (6)$$

and $H = \dot{a}/a$ is the Hubble parameter. Also the dot denotes a derivative with respect to cosmic time t . The Friedmann equations (4) and (5) can be rewritten in the standard form as [27]

$$H^2 + \frac{K}{a^2} = \frac{k^2}{3} \rho_t, \quad (7)$$

$$\dot{H} - \frac{K}{a^2} = -\frac{k^2}{2} (\rho_t + p_t). \quad (8)$$

Here, ρ_t and p_t are the total energy density and pressure defined as

$$\rho_t = \frac{\rho_m}{f_R} + \frac{\rho_R}{k^2}, \quad (9)$$

$$p_t = \frac{p_m}{f_R} + \frac{p_R}{k^2}, \quad (10)$$

and ρ_R and p_R are the energy density and pressure due to the curvature contribution defined as

$$\rho_R = \frac{1}{f_R} \left[-\frac{1}{2}(f - Rf_R) - 3H\dot{f}_R \right], \quad (11)$$

$$p_R = \frac{1}{f_R} \left[\frac{1}{2}(f - Rf_R) + 2H\dot{f}_R + \ddot{f}_R \right]. \quad (12)$$

Note that if $f(R) = R$, from Eqs. (11) and (12) we have $\rho_R = 0$ and $p_R = 0$ then Eqs. (7) and (8) transform to the usual Friedmann equations in the Einstein gravity.

The energy conservation laws are given by

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0, \quad (13)$$

$$\dot{\rho}_t + 3H(\rho_t + p_t) = 0. \quad (14)$$

Note that here the energy density and pressure due to the curvature contribution satisfies the following energy equation

$$\dot{\rho}_R + 3H(\rho_R + p_R) = \frac{\dot{f}_R}{f_R^2} \rho_m. \quad (15)$$

3 GSL in $f(R)$ -gravity

Here, we investigate the validity of the GSL in the framework of $f(R)$ -gravity. According to the GSL, entropy of the matter inside the horizon beside the entropy associated with the surface of horizon should not decrease during the time [12]. We consider a FRW universe filled only with the ordinary matter. We further assume the boundary of the universe to be enclosed by the dynamical apparent horizon with the Hawking temperature.

The dynamical apparent horizon in FRW universe is given by [28]

$$\tilde{r}_A = \left(H^2 + \frac{K}{a^2} \right)^{-1/2}, \quad (16)$$

which for the flat universe $K = 0$, it is same as the Hubble horizon, i.e. $\tilde{r}_A = H^{-1}$.

Following [12], the associated Hawking temperature on the apparent horizon is given by

$$T_A = \frac{1}{2\pi\tilde{r}_A} \left(1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right), \quad (17)$$

where $\frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} < 1$ ensure that the temperature is positive. Recently Cai et al. [29] proved that the apparent horizon of the FRW universe has an associated Hawking temperature (17). They also showed that this temperature can be measured by an observer with the Kodama vector inside the apparent horizon.

The entropy of the matter inside the horizon is given by the Gibbs' equation [18]

$$T_A dS_m = dE_m + p_m dV, \quad (18)$$

where $E_m = \rho_m V$ and $V = \frac{4\pi}{3}\tilde{r}_A^3$ is the volume containing the matter with the radius of the dynamical apparent horizon \tilde{r}_A . Taking time derivative of Eq. (18) and using the energy equation (13) one can find

$$T_A \dot{S}_m = (\rho_m + p_m)(\dot{V} - 3HV). \quad (19)$$

Substituting Eqs. (4) and (5) into the above relation gives

$$T_A \dot{S}_m = \frac{\tilde{r}_A^2}{2G} (\dot{\tilde{r}}_A - H\tilde{r}_A) \left(\frac{2K}{a^2} - 2\dot{H} + H \frac{d}{dt} - \frac{d^2}{dt^2} \right) f_R. \quad (20)$$

The horizon entropy in $f(R)$ -gravity is given by [16]

$$S_A = \frac{A f_R}{4G}, \quad (21)$$

where $A = 4\pi\tilde{r}_A^2$ is the area of the apparent horizon. Taking time derivative of Eq. (21) and using (17), one can get the evolution of the horizon entropy as

$$T_A \dot{S}_A = \frac{1}{4GH} (2H\tilde{r}_A - \dot{\tilde{r}}_A) \left(\frac{2\dot{\tilde{r}}_A}{\tilde{r}_A} + \frac{d}{dt} \right) f_R. \quad (22)$$

Now we can calculate the GSL due to different contributions of the matter and horizon. Adding Eqs. (20) and (22) and using the auxiliary relations

$$\begin{aligned}\dot{r}_A &= H\tilde{r}_A^3 \left(\frac{K}{a^2} - \dot{H} \right), \\ H\tilde{r}_A - \dot{r}_A &= H\tilde{r}_A^3 (\dot{H} + H^2), \\ 2H\tilde{r}_A - \dot{r}_A &= H\tilde{r}_A^3 \left(\frac{K}{a^2} + \dot{H} + 2H^2 \right),\end{aligned}\tag{23}$$

one can get the GSL in $f(R)$ -gravity as

$$\begin{aligned}T_A \dot{S}_{\text{tot}} &= \frac{1}{4G} \left(H^2 + \frac{K}{a^2} \right)^{-\frac{5}{2}} \left\{ 2H \left(\frac{K}{a^2} - \dot{H} \right)^2 f_R \right. \\ &\quad \left. + \left[\frac{K}{a^2} \left(\frac{K}{a^2} + \dot{H} + 3H^2 \right) - \dot{H}H^2 \right] \dot{f}_R + 2H(\dot{H} + H^2)\ddot{f}_R \right\},\end{aligned}\tag{24}$$

where $S_{\text{tot}} = S_m + S_A$. Equation (24) shows that the validity of the GSL, i.e. $T_A \dot{S}_{\text{tot}} \geq 0$, depends on the $f(R)$ -gravity model. For instance, in the Einstein gravity, i.e. $f(R) = R$, the GSL (24) yields

$$T_A \dot{S}_{\text{tot}} = \frac{1}{G} \left[\frac{H(\dot{H} - \frac{K}{a^2})^2}{2(H^2 + \frac{K}{a^2})^{5/2}} \right] \geq 0,\tag{25}$$

which shows that the GSL is always satisfied throughout the history of the universe. Using Eqs. (8) and (16), the above relation can be rewritten as

$$T_A \dot{S}_{\text{tot}} = 8\pi^2 G H \tilde{r}_A^5 (\rho_m + p_m)^2 \geq 0,\tag{26}$$

which is same as that obtained in [24].

4 GSL for a viable $f(R)$ model

Here, we would like to examine the validity of the GSL (24) for a viable $f(R)$ model given by

$$f(R) = R - \frac{\alpha}{R} + \beta R^2,\tag{27}$$

where α and β are two positive constants. This model is consistent with the cosmological and solar system tests [30]. It was also shown that the model (27) can predict the unification of the early time inflation and late time cosmic acceleration [31].

Note that to investigate the GSL (24) for model (27) we need to have the scale factor $a(t)$. The scale factor can be obtained from the first Friedmann equation (4). But before it one needs to determine the evolution of the matter from the energy equation (13). To do this if one takes the barotropic matter $p_m = \omega_m \rho_m$ with constant equation of state (EoS) parameter $\omega_m \geq 0$, then solving the energy equation (13) gives

$$\rho_m = \rho_{m0} a^{-3(1+\omega_m)}.\tag{28}$$

Substituting Eqs. (27) and (28) into (4) gives a complicated nonlinear differential equation for the scale factor that cannot be solved analytically. To avoid of this problem in what follows we investigate the validity of the GSL (24) for model (27) during the early inflation (large curvature) and late acceleration (small curvature) eras. In the intermediate epoch from strong

to low curvature R , the universe subsequently enters in the radiation and then matter dominated eras. In these two epochs, the last two terms in Eq. (27) have the same orders of magnitude as the first term. Therefore, model (27) recovers the Einstein gravity in which the GSL is satisfied (see Eq. 25). In what follows we further assume the universe to be spatially flat, i.e. $K = 0$, which is compatible with the observations [1].

4.1 The early inflation epoch

During the early inflationary phase of the universe, the scalar curvature R is large and model (27) behaves like

$$f(R) \simeq \beta R^2. \quad (29)$$

Here, we first consider the case of pure $f(R)$ -gravity in which the contribution of matter is neglected. In this case, inserting Eq. (29) into (4) gives the de Sitter scale factor

$$a(t) \propto e^{Ht}, \quad H = \text{constant}. \quad (30)$$

Also the deceleration parameter is obtained as

$$q = -1 - \frac{\dot{H}}{H^2} = -1. \quad (31)$$

Using Eqs. (29) and (30) the GSL (24) for $K = 0$ gives

$$T_A \dot{S}_{\text{tot}} = 0, \quad (32)$$

which corresponds to a reversible adiabatic expansion of the universe.

Now we investigate the more realistic case in which the $f(R)$ -gravity is to be minimally coupled with matter. Using Eqs. (28) and (29), solving Eq. (4) leads to the scale factor

$$a(t) \propto t^{\frac{4}{3(1+\omega_m)}}. \quad (33)$$

The corresponding Hubble and deceleration parameters are

$$H = \frac{4}{3(1+\omega_m)t}, \quad (34)$$

$$q = -1 + \frac{3(1+\omega_m)}{4}. \quad (35)$$

Here, due to having an accelerating universe, i.e. $q < 0$, from Eq. (35) we need to have $0 \leq \omega_m < 1/3$.

Using Eqs. (29) and (34), the GSL (24) for $K = 0$ yields

$$T_A \dot{S}_{\text{tot}} = \frac{9\beta(1-3\omega_m)(5-3\omega_m)}{4Gt^2}, \quad (36)$$

which shows that the GSL in the early inflation era is satisfied when $\omega_m \geq 5/3$ or $0 \leq \omega_m \leq 1/3$. However, the condition $\omega_m \geq 5/3$ should be avoided because it does not yield an accelerated inflationary phase of the universe (see Eq. 35). Note that $\omega_m = 0$ and $1/3$ correspond to the pressureless dust matter (or DM) and radiation, respectively.

4.2 The late acceleration era

In the late time epoch, the scalar curvature R is small. Hence, the only contribution that can reproduce the late time acceleration in model (27) is

$$f(R) \simeq -\alpha R^{-1}. \quad (37)$$

For the pure $f(R)$ -gravity case, i.e. $\rho_m \ll \rho_R$, solving Eq. (4) yields

$$a(t) \propto t^2, \quad (38)$$

and the Hubble parameter is obtained as

$$H = \frac{2}{t}. \quad (39)$$

Here, the deceleration parameter is $q = -1/2$. Using Eqs. (37) and (39), the GSL (24) for $K = 0$ leads to

$$T_A \dot{S}_{\text{tot}} = \frac{\alpha t^4}{1152 G} > 0, \quad (40)$$

which shows that the GSL holds for the case of pure $f(R)$ -gravity.

For the case of $f(R)$ -gravity coupled with matter, inserting Eqs. (28) and (37) into (4) gives the scale factor

$$a(t) \propto t^{\frac{-2}{3(1+\omega_m)}}, \quad (41)$$

which shows that the universe is shrinking in the presence of ordinary matter ($\omega_m \geq 0$). But if we change the arrow of time by $t \rightarrow t_s - t$, the expansion occurs with the inverse power law and at $t = t_s$, the size of the universe diverges. This modification yields the Hubble and deceleration parameters as

$$H = \frac{2}{3(1+\omega_m)(t_s - t)}, \quad t \leq t_s, \quad (42)$$

$$q = -1 - \frac{3(1+\omega_m)}{2}. \quad (43)$$

The above relation describes an expanding super-accelerating ($q < -1$) universe filled with ordinary matter ($\omega_m \geq 0$).

Using Eqs. (37) and (42) the GSL (24) for $K = 0$ yields

$$T_A \dot{S}_{\text{tot}} = \frac{243\alpha(1+\omega_m)^6(11+6\omega_m)(t_s - t)^4}{128G(7+3\omega_m)^2}, \quad (44)$$

which clarifies that the GSL is always satisfied for the case of $f(R)$ -gravity coupled with ordinary matter ($\omega_m \geq 0$).

5 Conclusions

Here, we studied the GSL of gravitational thermodynamics in the framework of $f(R)$ -gravity. Among other approaches related with a variety of DE models, a very promising approach to DE is related with the modified theories of gravity known as $f(R)$ -gravity, in which DE emerges from the modification of geometry. Modified gravity gives a natural unification of the early time inflation and late time acceleration thanks to different role of gravitational terms relevant

at large and at small curvature and may naturally describe the transition from deceleration to acceleration in the cosmological dynamics. We considered a FRW universe containing only the ordinary matter with positive constant EoS parameter ω_m . The boundary of the universe was assumed to be enclosed by the dynamical apparent horizon with the Hawking temperature. We examined the validity of the GSL for a viable model as $f(R) = R - \alpha/R + \beta R^2$ in the two regimes containing the early inflation and late acceleration eras. In the intermediate epoch containing the radiation and matter dominated eras, the GSL is always satisfied. We concluded that in the early time, the GSL for the pure $f(R)$ -gravity case (with no matter) behaves like a reversible adiabatic expansion of the de Sitter universe. Furthermore, for the case of $f(R)$ -gravity minimally coupled with matter the GSL is satisfied only for $0 \leq \omega_m \leq 1/3$. In the late time, for the case of pure $f(R)$ -gravity the GSL holds. Also for the case of $f(R)$ -gravity coupled with ordinary matter the GSL is always satisfied.

Acknowledgements

The work of K. Karami has been supported financially by Research Institute for Astronomy and Astrophysics of Maragha (RIAAM) under research project No. 1/2342.

References

- [1] A.G. Riess, et al., *Astron. J.* **116**, 1009 (1998);
S. Perlmutter, et al., *Astrophys. J.* **517**, 565 (1999);
P. de Bernardis, et al., *Nature* **404**, 955 (2000);
S. Perlmutter, et al., *Astrophys. J.* **598**, 102 (2003).
- [2] T. Padmanabhan, *Phys. Rep.* **380**, 235 (2003);
P.J.E. Peebles, B. Ratra, *Rev. Mod. Phys.* **75**, 559 (2003);
C.G. Tsagas, A. Challinor, R. Maartens, *Phys. Rept.* **465**, 61 (2008);
M. Li, X.D. Li, S. Wang, Y. Wang, *Commun. Theor. Phys.* **56**, 525 (2011).
- [3] V. Sahni, *Class. Quantum Grav.* **19**, 3435 (2002);
E.J. Copeland, M. Sami, S. Tsujikawa, *Int. J. Mod. Phys. D* **15**, 1753 (2006);
T. Padmanabhan, *Gen. Relativ. Gravit.* **40**, 529 (2008).
- [4] T.P. Sotiriou, V. Faraoni, *Rev. Mod. Phys.* **82**, 451 (2010);
S. Nojiri, S.D. Odintsov, *Phys. Rept.* **505**, 59 (2011).
- [5] S. Capozziello, *Int. J. Mod. Phys. D* **11**, 483 (2002).
- [6] S. Nojiri, S.D. Odintsov, *Phys. Rev. D* **74**, 086005 (2006);
S. Nojiri, S.D. Odintsov, *J. Phys. Conf. Ser.* **66**, 012005 (2007);
S. Nojiri, S.D. Odintsov, D. Saez-Gomez, arXiv:0908.1269.
- [7] A.A. Starobinsky, *Phys. Lett. B* **91**, 99 (1980);
R. Kerner, *Gen. Relativ. Gravit.* **14**, 453 (1982);
J. Barrow, A. Ottewill, *J. Phys. A* **16**, 2757 (1983);
V. Faraoni, *Phys. Rev. D* **74**, 023529 (2006);
H.J. Schmidt, *Int. J. Geom. Math. Phys.* **4**, 209 (2007);
W. Hu, I. Sawicki, *Phys. Rev. D* **76**, 064004 (2007).

- [8] S. Nojiri, S.D. Odintsov, Gen. Relativ. Gravit. **36**, 1765 (2004);
S.D. Odintsov, S. Nojiri, Mod. Phys. Lett. A **19**, 627 (2004);
M.C.B. Abdalla, S. Nojiri, S.D. Odintsov, Class. Quantum Grav. **22**, L35 (2005).
- [9] S. Nojiri, S.D. Odintsov, Phys. Lett. B **576**, 5 (2003);
S.M. Carroll, et al., Phys. Rev. D **70**, 043528 (2004);
S. Capozziello, S. Nojiri, S.D. Odintsov, Phys. Lett. B **634**, 93 (2006);
S. Nojiri, S.D. Odintsov, D. Sáez-Gómez, Phys. Lett. B **681**, 74 (2009);
Y. Bisabr, Phys. Scr. **80**, 045902 (2009);
K. Karami, M.S. Khaledian, JHEP **03**, 086 (2011);
A. Khodam-Mohammadi, P. Majari, M. Malekjani, Astrophys. Space Sci. **331**, 673 (2011).
- [10] G.R. Bengochea, R. Ferraro, Phys. Rev. D **79**, 124019 (2009);
K. Karami, A. Abdolmaleki, arXiv:1009.2459.
- [11] Y. Sobouti, Astron. Astrophys. **464**, 921 (2007).
- [12] R.G. Cai, S.P. Kim, JHEP **02**, 050 (2005).
- [13] M. Akbar, R.G. Cai, Phys. Lett. B **635**, 7 (2006);
M. Akbar, R.G. Cai, Phys. Lett. B **648**, 243 (2007).
- [14] M. Akbar, R.G. Cai, Phys. Rev. D **75**, 084003 (2007).
- [15] A. Sheykhi, JCAP **05**, 019 (2009).
- [16] R.M. Wald, Phys. Rev. D **48**, 3427 (1993);
G. Cognola, E. Elizalde, S.Nojiri, S.D. Odintsov, S. Zerbini, JCAP **02**, 010 (2005).
- [17] R.X. Miao, M. Li, Y.G. Miao, JCAP **11**, 033 (2011);
K. Karami, A. Abdolmaleki, JCAP **04**, 007 (2012).
- [18] G. Izquierdo, D. Pavón, Phys. Lett. B **639**, 1 (2006).
- [19] H. Mohseni Sadjadi, Phys. Rev. D **73**, 063525 (2006);
H. Mohseni Sadjadi, Phys. Rev. D **76**, 104024 (2007);
H. Mohseni Sadjadi, Phys. Lett. B **645**, 108 (2007).
- [20] J. Zhou, B. Wang, Y. Gong, E. Abdalla, Phys. Lett. B **652**, 86 (2007).
- [21] Y. Gong, B. Wang, A. Wang, Phys. Rev. D **75**, 123516 (2007);
Y. Gong, B. Wang, A. Wang, JCAP **01**, 024 (2007).
- [22] M. Jamil, E.N. Saridakis, M.R. Setare, JCAP **11**, 032 (2010).
- [23] A. Sheykhi, B. Wang, Phys. Lett. B **678**, 434 (2009);
A. Sheykhi, B. Wang, Mod. Phys. Lett. A **25**, 1199 (2010);
A. Sheykhi, Phys. Rev. D **81**, 104011 (2010);
A. Sheykhi, Eur. Phys. J. C **69**, 265 (2010);
A. Sheykhi, Class. Quantum Grav. **27**, 025007 (2010).

- [24] K. Karami, JCAP **01**, 015 (2010);
 K. Karami, S. Ghaffari, Phys. Lett. B **685**, 115 (2010);
 K. Karami, S. Ghaffari, Phys. Lett. B **688**, 125 (2010);
 K. Karami, S. Ghaffari, M.M. Soltanzadeh, Class. Quantum Grav. **27**, 205021 (2010);
 K. Karami, S. Ghaffari, M.M. Soltanzadeh, Astrophys. Space Sci. **331**, 309 (2011);
 K. Karami, A. Sheykhi, N. Sahraei, S. Ghaffari, Europhys. Lett. **93**, 29002 (2011);
 K. Karami, A. Abdolmaleki, N. Sahraei, S. Ghaffari, JHEP **08**, 150 (2011).
- [25] N. Radicella, D. Pavón, Phys. Lett. B **691**, 121 (2010).
- [26] K. Bamba, C.Q. Geng, JCAP **11**, 008 (2011).
- [27] S. Capozziello, V.F. Cardone, S. Carloni, A. Troisi, Int. J. Mod. Phys. D **12** 1969, (2003);
 S. Capozziello, V.F. Cardone, A. Troisi, Phys. Rev. D **71**, 043503 (2005).
- [28] E. Poisson, W. Israel, Phys. Rev. D **41**, 1796 (1990);
 S.A. Hayward, Phys. Rev. D **53**, 1938 (1996);
 Y.G. Gong, A. Wang, Phys. Rev. Lett. **99**, 211301 (2007).
- [29] R.G. Cai, L.M. Cao, Y.P. Hu, Class. Quantum Grav. **26** 155018 (2009).
- [30] S. Nojiri, S.D. Odintsov, Int. J. Geom. Meth. Mod. Phys. **4**, 115 (2007).
- [31] S. Nojiri, S.D. Odintsov, Phys. Rev. D **68**, 123512 (2003).